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NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 01/23/91

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roject Director LEWIS F L School/Lab ELEC ENGR

ponsor NATL SCIENCE FOUNDATION/GENERAL

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Project Director	Y
Administrative Network Representative	Y
GTRI Accounting/Grants and Contracts	Y
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Research Security Services	N
Reports Coordinator (OCA)	Y
GTRC	Y
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E-21-675

STRUCTURE AND OUTPUT FEEDBACK

IN SINGULAR SYSTEMS

NSF Grant ECS-8805932

Annual Progress Report for
Sept. 1988 - April 1989

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APPENDIX B. RECENT PAPERS:

1. F.L. Lewis, G. Beauchamp, K. Özçaldiran, and R.P. Malhamé, "Large-scale dynamical interconnections of stochastic singular systems," Circuits, Systems, and Signal Proc., submitted.

2. G. Beauchamp and F.L. Lewis, "On the analysis and solution of two-dimensional boundary-value discrete singular systems," Proc. IEEE Conf. Decision and Control, Tampa, FL, Dec. 1989, submitted.

3. A. Karamancioḡlu and F.L. Lewis, "A geometric approach to 2-D singular systems using the Roesser model," in preparation.

4. F.L. Lewis and V.L. Syrmos, A geometric theory for derivative feedback," IEEE Trans. Automat. Control, submitted.

5. V.L. Syrmos and F.L. Lewis, "A geometric theory of invariant, partitioned, and deflating subspaces for singular systems," SIAM J. Control and Opt., submitted.

6. F.L. Lewis and D.W. Fountain, "Generalized notions in geometry and duality," Proc. American Control Conf., June 1989.

I. INTRODUCTION

There were two goals in the proposal. They were to:

1. Study the properties and extensions of the singular system structure algorithm (SSSA)

2. Study proportional-plus-derivative (PD) output feedback in state-variable and singular systems.

Significant progress has been made during the first year in both areas.

A Summary of Activities and Research appears in Appendix A.

As far as the progress under Goal 1 goes, new results include

1. an algorithm to test for the absence of input derivatives in stochastic singular systems,

2. a boundary recursion for 2-D singular systems,

3. a structure algorithm for 2-D singular systems.

Results under Goal 2 include

4. a geometric theory for derivative (D) feedback,

5. new definitions of deflating subspaces and partitioned subspaces,

6. a geometric theory for PD feedback,

7. new results in reachability, observability, and duality for discrete singular system.

In addition, miscellaneous results include preliminary work on

8. a geometric theory for 2-D singular systems using the Roesser model.

We shall briefly discuss the results. Some recent relevant papers are enclosed as Appendix B; the references are to these attached papers.

II. TEST FOR THE ABSENCE OF INPUT DERIVATIVES

Consider the singular system

$$E\dot{x} = Ax + Bu \tag{2.1}$$

with E square and generally singular, and u(t) a white noise process.

It is well known that in singular systems, the semistate x(t) generally contains derivatives of the input u(t). If u(t) is a white noise process, derivatives are undesirable. If no derivatives of u(t) appear in x(t), we call (2.1) <u>stochastically well-defined</u>.

In [1] attached we have provided a test for the absence of derivatives of the input. The test is based on the following modification of the SSSA.

<u>Algorithm 2.1</u>

Perform

$$T_k \begin{bmatrix} E_k & A_k & B_k & B_k' \\ -\underline{A}_k & 0 & 0 & \underline{B}_k \end{bmatrix} \begin{matrix} r_k \\ t_k \end{matrix} = \begin{bmatrix} E_{k+1} & A_{k+1} & B_{k+1} & B_{k+1}' \\ 0 & \underline{A}_{k+1} & \underline{B}_{k+1} & \underline{B}_{k+1}' \end{bmatrix} \begin{matrix} r_{k+1} \\ \end{matrix},$$

$$k= 0,1,\ldots, L-1. \tag{2.2}$$

where T_k is a constant orthogonal transformation, E_{k+1} has full row rank r_{k+1}, and the initial matrices are $E_0= E$, $A_0= A$, $B_0= B$, $B_0'=0$, and both, $\underline{A}_0$, and $\underline{B}_0$ are null matrices. Define L as the first value of k+1 for either which $r_{k+1}= n$ or $\underline{A}_{k+1}= 0$. ■

The next theorem provides the test.

<u>Theorem 2.2</u>

Let (2.1) be regular. Then (2.1) is stochastically well-defined if and only if

$$\operatorname{rank}\begin{bmatrix} E_k & B_k' \\ -\underline{A}_k & \underline{B}_k \end{bmatrix} = \operatorname{rank}\begin{bmatrix} E_k \\ -\underline{A}_k \end{bmatrix}; \; k=1,2,\ldots, L-1. \tag{2.3}$$

■

What this ammounts to is a test for <u>un</u>controllability at infinity. In fact, (2.1) is stochastically well-defined if and only if the only chains reachable at infinity are trivial (i.e.

have length one).

III. BOUNDARY RECURSION FOR 2-D SINGULAR SYSTEMS

In [2] attached the boundary recursion of Luenberger was extended to 2-D systems satisfying the Fornasini-Marchesini model

$$Ex_{i+1,j+1} = A_0 x_{i,j} + A_1 x_{i+1,j} + A_2 x_{i,j+1} + Bu_{i,j} \tag{3.1a}$$

$$y_{i,j} = Cx_{i,j} \;;\; i = 0,\ 1,\ \ldots,\ N_1-1,\ j = 0,\ 1,\ \ldots,\ N_2-1 \tag{3.1b}$$

where $x \in \mathbf{R}^n$ is the semistate vector, $u \in \mathbf{R}^m$ is the input vector, and $y \in \mathbf{R}^p$ is the output vector. E is a constant square matrix of dimension n, generally singular, A_0, A_1, A_2, B, and C are constant matrices of appropriate dimensions, and N_1 and N_2 are given fixed integers. The region of interest in the (i,j)-plane is the 2-D region bounded by the rectangle $[0,N_1]\mathrm{x}[0,N_2]$.

Ordering the 2-D semistate $x_{i,j}$ by increasing (i,j) in odometer order (i.e. by columns), we may write all the dynamical equations (3.1) over the entire region of interest in the form

$$\left[\begin{array}{ccc|ccc|ccc}
-A_0 & -A_2 & & -A_1 & E & & & & \\
 & -A_0 & -A_2 & & -A_1 & E & & & \\
 & & \ddots\ \ddots & & & \ddots\ \ddots & & & \\
 & & -A_0\ \ -A_2 & & & -A_1\ \ E & & & \\
\hline
 & & & -A_0 & -A_2 & & -A_1 & E & \\
 & & & & -A_0 & -A_2 & & -A_1 & E \\
 & & & & & \ddots\ \ddots & & & \ddots\ \ddots \\
 & & & & & -A_0\ \ -A_2 & & & -A_1\ \ E
\end{array}\right]
\left[\begin{array}{c}
x_{00} \\ x_{01} \\ x_{02} \\ \vdots \\ x_{0,N2} \\ \hline
x_{10} \\ x_{11} \\ x_{12} \\ \vdots \\ x_{1,N2} \\ \hline
x_{20} \\ x_{21} \\ x_{22} \\ \vdots \\ x_{2,N2}
\end{array}\right] =$$

$$
\left[\begin{array}{ccc|ccc}
B & & & & & \\
& \ddots & & & & \\
& & B & & & \\
\hline
& & & B & & \\
& & & & \ddots & \\
& & & & & B
\end{array}\right]
\left[\begin{array}{c}
u_{00} \\
u_{01} \\
\vdots \\
\hline
u_{0,N2-1} \\
u_{10} \\
u_{11} \\
\vdots \\
\hline
u_{1,N2-1}
\end{array}\right] . \qquad (3.2a)
$$

The output equation (3.1b) yields

$$
\left[\begin{array}{c}
y_{00} \\
y_{01} \\
\vdots \\
y_{0,N2} \\
\hline
y_{10} \\
y_{11} \\
\vdots \\
y_{1,N2} \\
\hline
y_{20} \\
y_{21} \\
\vdots \\
y_{2,N2}
\end{array}\right]
=
\left[\begin{array}{ccc|ccc|ccc}
C & & & & & & & & \\
& \ddots & & & & & & & \\
& & C & & & & & & \\
\hline
& & & C & & & & & \\
& & & & \ddots & & & & \\
& & & & & C & & & \\
\hline
& & & & & & C & & \\
& & & & & & & \ddots & \\
& & & & & & & & C
\end{array}\right]
\left[\begin{array}{c}
x_{00} \\
x_{00} \\
\vdots \\
x_{0\ N2} \\
\hline
x_{10} \\
x_{11} \\
\vdots \\
x_{1,N2} \\
\hline
x_{20} \\
x_{21} \\
\vdots \\
x_{2,N2}
\end{array}\right] . \qquad (3.2b)
$$

This system of equations may be written as

$$\begin{bmatrix} -F & G & & & & \\ & -F & G & & & \\ & & \cdot & \cdot & & \\ & & & \cdot & \cdot & \\ & & & & \cdot & \cdot \\ & & & & -F & G \end{bmatrix} \begin{bmatrix} \underline{x}_0 \\ \underline{x}_1 \\ \cdot \\ \cdot \\ \cdot \\ \underline{x}_{N1} \end{bmatrix} = \begin{bmatrix} \underline{B} & & & & \\ & \underline{B} & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ & & & & \underline{B} \end{bmatrix} \begin{bmatrix} \underline{u}_0 \\ \underline{u}_1 \\ \cdot \\ \cdot \\ \cdot \\ \underline{u}_{N1-1} \end{bmatrix} \tag{3.3a}$$

$$\begin{bmatrix} \underline{y}_0 \\ \underline{y}_1 \\ \cdot \\ \cdot \\ \cdot \\ \underline{y}_{N1} \end{bmatrix} = \begin{bmatrix} \underline{C} & & & & \\ & \underline{C} & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \cdot \\ & & & & \underline{C} \end{bmatrix} \begin{bmatrix} \underline{x}_0 \\ \underline{x}_1 \\ \cdot \\ \cdot \\ \cdot \\ \underline{x}_{N1} \end{bmatrix} \tag{3.3b}$$

where the matrices F, G, $\underline{B}$, and $\underline{C}$, and the vectors $\underline{x}_i$, $\underline{u}_i$, and $\underline{y}_i$ are defined according to the block structure of (3.2).

In terms of these constructions we are able to prove conditions for the existence and uniqueness of the solution in terms of the 2-D matrix pencil

$$P(z_1,z_2)= [z_1z_2E - z_1A_1 - z_2A_2 - A_0]. \tag{3.4}$$

The boundary conditions for (3.1) are restrictions on the semistates around the rectangular boundary of the region $[0,N_1]x[0,N_2]$ of the form

$$C_{i,0}x_{i,0} + C_{i,N2}x_{i,N2}= c^h_i \;, \; i= 1,\ 2,\ \ldots,\ N_1; \tag{3.5a}$$

$$C_{0,j}x_{0,j} + C_{N1,j}x_{N1,j}= c^v_j \;, \; j= 0,\ 1,\ \ldots,\ N_2 \tag{3.5b}$$

where both $[C_{i,0}\ ,\ C_{i,N2}]$ and $[C_{0,j}\ ,\ C_{N1,j}]\epsilon\ \mathbf{R}^{n\times 2n}$ are of full row rank and c^h_i, $c^v_j\epsilon\ \mathbf{R}^n$ are given constant vectors.

These boundary conditions are similar, for instance, to the boundary conditions of the heat equation over a rectangular plane, which are given at the four edges of the plane.

We can take advantage of the block structure of (3.3) to provide an algorithm for solving for $x_{i,j}$ in (3.1) with the specified boundary conditions. This algorithm is just a 2-D or double-indexed version of Luenberger's 1-D Boundary Recursion.

Also under investigation are the extension of the SSSA to 2-D systems as well as a geometric theory for singular 2-D systems using the Roesser model, which is a generalization of (3.1). Since these results are not yet complete, we only refer here to [3] attached, which outlines some notions in the latter topic.

IV. GEOMETRIC THEORY FOR DERIVATIVE FEEDBACK

In [4] attached we have laid the foundations for a geometric theory for derivative feedback. Some of the important notions are summarized here.

Consider (2.1), which we assume to be regular (i.e. $|sE-A| \neq 0$), where u(t) is now a (deterministic) control input defined by the D feedback

$$u = -k\dot{x}. \tag{4.1}$$

The closed-loop system is

$$(E+BK)\dot{x} = Ax + Bu. \tag{4.2}$$

In the state-space case E=I, it is not guaranteed that (4.2) will be in state-space form since $|I+BK|$ may be zero. Thus, the state-variable systems are not closed under D feedback. This accounts for the fact that there has been to date no geometric theory for derivative feedback.

The lack of closure of the state-variable systems under D feedback is a fairly serious deficiency, since it is well known from classical control theory that D feedback is often extremely useful.

Define $\mathbf{S} \subset \mathbf{R}^n$ as an <u>(E,A,B)-invariant subspace</u> for (2.1) if it satisfies

$$E\mathbf{S} \subset A\mathbf{S} + R(B). \tag{4.3}$$

This is equivalent to the existence of matrices F and G such that

$$ES = ASF - BG, \tag{4.4}$$

with S a basis for $\mathbf{S}$; (4.4) is a generalized Lyapunov or Sylvester equation.

The following theorem shows the meaning of $\mathbf{S}$.

<u>Theorem 4.1</u>

$\mathbf{S} \subset \mathbf{R}^n$ is an (E,A,B)-invariant subspace of (2.1) if and only if, for any $x(0^-) \in \mathbf{S}$, there exists an input u(t) such that

1. $x(t) \in \mathbf{S}$ for $t \geq 0$,

2. The Laplace transforms of $u(t)$ and $\dot{x}(t)$ have no poles at the origin.

■

In [4] we show how to compute the supremal (E,A,B)-invariant subspace using a subspace recursion as well as a variant of the SSSA.

The notions of the anticausal system pencil $(E-zA)$ and the anticausal controllability pencil

$$C_d(z)= [E-zA \quad B] \tag{4.5}$$

were introduced and shown to be relevant in the study of D feedback. Some connections were drawn with the work of Karcanias and also that of Schumacher. (The nomenclature "anticausal" derives from the fact that, in a discrete-time system reversing the roles of E and A amounts to writing $Ax_k= Ex_{k+1} - Bu_k$.)

The next results show the relation between the (E,A,B)-invariant subspaces and what is possible using D feedback. They extend some of the results of Wonham to the case of D feedback.

<u>**Theorem 4.2**</u>

$\mathbf{S}$ satisfies (4.3) if and only if there exists a K such that

$$(E+BK)\mathbf{S} \subset A\mathbf{S} \ . \tag{4.6}$$

■

The D feedback required to assign $\mathbf{S}$ as a closed-loop (E,A)-invariant subspace satisfying (4.6) is simply given in terms of the solution to the Lyapunov equation (4.4). Indeed, any K satisfying

$$KS=G \tag{4.7}$$

suffices. We have also demonstrated that using such a feedback, the spectrum assigned on $\mathbf{S}$ in the closed-loop system is nothing but $\sigma(F)$.

This generalizes the work of Moore to D feedback, for (4.4) is nothing but his eigenstructure assignment equation in streamlined form. The columns of S are the closed-loop eigenvectors. To see this, denote the i-th column of a matrix M as M_i. Then, if $F= \mathrm{diag}\{\lambda_i\}$, (4.4) and (4.7) become

$$(E-\lambda_i A)S_i= BG_i \tag{4.8}$$

$$KS_i= G_i . \tag{4.9}$$

Note the appearance of the anticausal system pencil in (4.8).

Our approach has an advantage in that the closed-loop eigenstructure need not be simple. That is, F may be a matrix with a nondiagonal Jordan form.

The remainder of [4] provides a numerical procedure for finding solutions (S,F,G) to (4.4) that assign a desired closed-loop eigenstructure.

V. GEOMETRIC THEORY FOR PD FEEDBACK

In [5] attached we have extended the work in Section IV. to the more general case of PD feedback.

We introduce the following notions for the system (2.1), with $x \in \mathbf{R}^n$.

A *partition* of $\mathcal{S} \subset \mathbf{R}^n$ into two subspaces $\mathcal{U}, \mathcal{V} \subset \mathcal{S}$ is defined as

(i) $A\mathcal{V} \subset E\mathcal{V} + \mathcal{B}$ (5.1a)

(ii) $E\mathcal{U} \subset A\mathcal{U} + \mathcal{B}$ (5.1b)

(iii) $\mathcal{S} = \mathcal{U} \oplus \mathcal{V}$ (5.1c)

■

The subspaces $\mathcal{U},\mathcal{V}$ are recognized as $(E,A,\mathcal{B})$ and $(A,E,\mathcal{B})$-invariant subspaces respectively. Every subspace $\mathcal{S}$ can always be partitioned in this fashion. We shall call $\mathcal{S}$ a *partitioned subspace* and denote it as $\mathcal{S} = \mathcal{U} \odot \mathcal{V}$.

A *regular partition* of $\mathcal{S}$ into two subspaces $\mathcal{U},\mathcal{V} \subset \mathcal{S}$ is defined as a partition of $\mathcal{S}$ with the two additional properties

(i) $\dim(E\mathcal{V}) = \dim\mathcal{V}$ (5.2a)

(ii) $\dim(A\mathcal{U}) = \dim\mathcal{U}$ (5.2b)

Choosing V and U as bases for $\mathcal{V}$ and $\mathcal{U}$ respectively, (5.1a) and (5.1b) are equivalent respectively to

$$AV = EVF_1 + BG_1 \tag{5.3}$$

$$EU = AUF_2 + BG_2 \tag{5.4}$$

for some F_1, G_1, F_2, G_2. These are *generalized Lyapunov or Sylvester equations*.

The following theorem points out some of the properties enjoyed by a partition.

Theorem 2.3

$S = U \oplus V$ is a partitioned subspace if and only if for any $x(0^-)$ there exists an input u(t) such that :

(i) $x(t) \in S$ for $t \geq 0$, for some u(t).
(ii) The Laplace transforms of x(t), u(t) restricted to V are strictly proper.
(iii) The Laplace transforms of $\dot{x}(t)$, u(t) restricted to U have no poles at the origin.

■

Note: For comparison of (ii) and (iii), note that a strictly proper transfer function has no poles at infinity.

$S \subset \mathbf{R}^n$ is an (E,A,B)-deflating subspace if

$$\dim(ES + AS + BG) = \dim S \tag{5.5}$$

for some G. This generalizes the well-known idea of deflating subspace by including the effect of the control input.

In [4] we show several results in connection with these new definitions. We also give the algebraic and dynamical properties of these geometrical objects.

The important point is that we use these concepts to characterize the possible closed-loop structure under the PD feedback

$$u = -K_1 x + K_2 \dot{x}, \tag{5.6}$$

which results in the closed-loop system

$$(E + BK_2)\dot{x} = (A + BK_1). \tag{5.7}$$

Specifically, we show that corresponding to every regular partition $S = U \odot V$, there exist K_1 and K_2 such that the closed-loop system has a forward/backward decomposition on S along U + V. Moreover, the required feedback gains are easily determined in terms of the solutions to the Lyapunov equations (5.3), (5.4) by

$$[G_1 \; 0] = K_1[V \; U] \tag{5.8a}$$

$$[0 \; G_2] = K_2[V \; U] \tag{5.8b}$$

The closed-loop spectrum of (5.7) is determined on $S = U \odot V$ by $\sigma(F_1) \cup \sigma(F_2^{-1})$.

Also discussed is a technique for determining solutions to the

Lyapunov equations that assign a specified closed-loop eigenstructure on a given S using PD feedback.

VI. REACHABILITY, OBSERVABILITY, AND DUALITY

In [6] attached is provided a more complete and pleasing theory of duality for singular systems than is currently available.

In the work of Cobb, a duality theory was provided in terms of "slow" and "fast" subspaces defined in terms of the Weierstrass form. Our theory, on the other hand, is in terms of subspaces defined directly in terms of the original system matrices. These subspaces have the additional advantage of possessing straightforward properties in terms of the system dynamics, as well as being convenient to compute using the SSSA.

In this work we consider the linear, time-invariant, discrete-time singular system

$$Ex_{i+1} = Ax_i + Bu_i \tag{6.1}$$

$$y_i = Cx_i \tag{6.2}$$

with $i\epsilon[-\mu,M-1]$ where μ is the index of nilpotency of the matrix E and M is a fixed integer with $M\geq\mu$. We assume that E is square and (6.1) is regular. The domain of E and A is denoted by **X**.

The reachability properties are defined in terms of the subspaces

$$\mathbf{V}_{k+1}= A^{-1}(E\mathbf{V}_k + \text{Im } B) \text{ , with } \mathbf{V}_0= \mathbf{H}_I \tag{6.3}$$

$$\mathbf{R}^a_{k+1}= E^{-1}(A\mathbf{R}^a_k + \text{Im } B) \text{ , with } \mathbf{R}^a_0= \mathbf{H}_F \tag{6.4}$$

$$\mathbf{R}_k = \mathbf{R}^a_k \cap \mathbf{V}_k. \tag{6.5}$$

with $\mathbf{H}_I$ and $\mathbf{H}_F$ respectively Wong's initial and final manifolds, which are easy to compute using the SSSA.

The next two results give the dynamical significance of $\mathbf{V}_k$, $\mathbf{R}^a_k$, and $\mathbf{R}_k$.

Theorem 6.1

1. The sequence of subspaces $\mathbf{V}_k$ has the following dynamical interpretation:

   $E\mathbf{V}_k$ = {Exϵ**X**: there exists a control sequence u_0, u_1, ..., u_{k-1}, and a semistate sequence x_0, x_1, ..., x_k, satisfying (6.1) for i=0, ..., k-1 and such that x_0=x}.

2. The sequence of subspaces R^a_k has the following dynamical interpretation:

R^a_k = {$x \epsilon X$: there exists a control sequence u_0, u_1, ..., u_{k-1}, and a semistate sequence x_{-q}, ..., x_0, x_1, ..., x_k, satisfying (6.1) for i=0, ..., k-2 and such that $x_{-q}=0$ for some q and $x_k=x$}.

3. For $k \geq \mu$, the sequence R_k gives the reachable subspace:

R_k = {$x \epsilon X$: there exists a control sequence u_0, u_1, ..., $u_{k+\mu-1}$, and a semistate sequence x_0, x_1, ..., x_k, satisfying (6.1) for i=0, ..., k-1 and such that $x_k=x$}

■

These notions naturally extend the corresponding results for state-variable systems. In fact, R^a_k are Willems' "almost reachability subspaces". In continuous time they correspond to the requirement for impulses in the control input, while in the discrete-time case they require noncausal semistate sequences. By intersecting R^a_k with V_k we cure the problem of the requirement for impulsive inputs, or the noncausal problem in the discrete case, and obtain the usual reachability subspaces R_k.

These results could also be considered as extending the work of Molinari to singular systems.

Our study of observability relies on the subspaces defined by

$$\underline{S}_{k+1} = A(E^{-1}\underline{S}_k \cap \text{Ker } C) \text{ , with } \underline{S}_0 = AH_F \tag{6.6}$$

$$\underline{U}_{k+1} = E(A^{-1}\underline{U}_k \cap \text{Ker } C) \text{ , with } \underline{U}_0 = EH_I \tag{6.7}$$

$$T_k = A^{-1}\underline{S}_k + (E^{-1}\underline{U}_k \cap \text{Ker } C). \tag{6.8}$$

The underbar denotes subspaces in the codomain of E and A.

The next theorem gives the dynamical significance of these subspaces.

<u>**Theorem 6.2**</u>

1. The subspaces $\underline{S}_k$ have the following dynamical significance:

$A^{-1}\underline{S}_k$ = {$x \epsilon X$: with no input, there exists a semistate sequence $x_{-\mu}$, $x_{-(\mu-1)}$, ..., x_0 with $y_{-(k-1)}$, ..., $y_0 = 0$ and $x_0=x$}.

2. For k>0, the subspaces $\underline{U}_k$ have the following dynamical significance:

$E^{-1}\underline{U}_k \cap$ **Ker** C= {$x \epsilon$ **X**: with no input, there exists a semi-state sequence which is zero for $i<0$ with $y_0, \ldots, y_{k-1}$ = 0 and x_0=x}.

3. For k>0, the subspaces $\mathbf{T_k}$ have the following interpretation:

$\mathbf{T_k}$= {$x \epsilon$ **X**: with no input, there exists a semistate sequence $x_{-\mu}, \ldots, x_M$ with $y_{-(k-1)}, \ldots, y_{k-1} = 0$, and x_0=x}.

■

The dual system for (6.1)-(6.2) is defined by

$$E^T x_{i+1} = A^T x_i + C^T u_i \tag{6.9}$$

$$y_i = B^T x_i. \tag{6.10}$$

We denote by superscript "D" the subspaces defined in terms of the dual system.

The remainder of [6] shows that the following duality relations hold:

$$\underline{S}_k = (V_k^D)^{\perp} \tag{6.11}$$

$$\underline{U}_k = ((R^a_k)^D)^{\perp}. \tag{6.12}$$

APPENDIX A
SUMMARY OF ACTIVITIES AND RESEARCH

I. ITEMS ACCEPTED

Invited Journal and Conference Papers

1. F.L. Lewis, G. Beauchamp, and V.L. Syrmos, "Some useful aspects of the infinite structure in singular systems," Proc. MTNS, June 1989.

2. F.L. Lewis and D.W. Fountain, "Generalized notions in geometry and duality," Proc. American Control Conf., June 1989.

3. F.L. Lewis, G. Beauchamp, and B. Mertzios, "Recent results in 2-D singular systems," Proc. IFAC Workshop on System Structure and Control, Prague, Czechoslovakia, Sept. 1989.

4. F.L. Lewis, "A tutorial on singular systems," Proc. IFAC Workshop on System Structure and Control, Prague, Czechoslovakia, Sept. 1989.

5. F.L. Lewis, "Computational geometry for design in singular systems," IMACS Trans. Scientific Comp., to appear 1989.

6. F.L. Lewis and B.G. Mertzios, ed., Circuits, Systems, and Signal Proc., Special Issue on "Recent Advances in Singular Systems", to appear 1989.

7. B.G. Mertzios and F.L. Lewis, "Fundamental matrix of discrete singular systems," Circuits, Systems, and Signal Proc., to appear, 1989.

Journal Papers

1. B.G. Mertzios and F.L. Lewis, "An algorithm for the computation of the transfer function matrix of generalized 2-D systems," Circuits, Systems, and Signal Proc., vol. 7, no. 4, pp. 459-466, 1988.

2. F.L. Lewis and K. Özçaldiran, "Geometric structure and feedback in singular systems," IEEE Trans. Automat. Control, vol. 34, no. 4, pp. 450-454, April 1989.

3. K. Özçaldiran and F.L. Lewis, "Generalized reachability subspaces for singular systems," SIAM J. Control and Opt., to appear, 1989.

4. F.L. Lewis, V.G. Mertzios, G. Vachtsevanos, and M.A. Christodoulou, "Analysis of bilinear systems using Walsh functions," IEEE Trans. Automat. Control, to appear, 1989.

5. F.L. Lewis, M.A. Christodoulou, B.G. Mertzios, and K. Özçaldiran, "Chained aggregation of singular systems," IEEE Trans. Automat. Control, to appear, 1989.

6. F.L. Lewis and B.G. Mertzios, "On the analysis of discrete linear time-invariant singular systems," IEEE Trans. Automat. Control, to appear, 1989.

7. F.L. Lewis, "Geometric design techniques for feedback and observers in singular systems," Automatica, accepted pending revision, 1989.

8. K. Özçaldiran and F.L. Lewis, "On the regularizability of singular systems," IEEE Trans. Automat. Control,

II. ITEMS SUBMITTED AND IN PREPARATION

Books in Progress

1. F.L. Lewis, K. Ozcaldiran, and B.G. Mertzios, Singular Systems, Prentice-Hall, International Series in Systems and Control Engineering, contract under negotiation, to appear, 1991.

Proposals

1. A.C. Pugh, F. L. Lewis, and G. Hayton, NATO Workshop on Singular Systems, Florence, Sept. 1990, proposal in preparation.

Journal Papers

1. F.L. Lewis, B.G. Mertzios, and W. Marszalek, "Walsh function analysis of linear and bilinear discrete singular systems," Int. J. Control, submitted.

2. F.L. Lewis, B.G. Mertzios, and W. Marszalek, "Walsh function analysis of 2-D generalized systems," IEEE Trans. Acoustics, Speech, and Signal Proc., submitted.

3. F.L. Lewis, W. Marszalek, and B.G. Mertzios, "Walsh function analysis of 2-D generalized continuous systems," IEEE Trans. Automat. Control, submitted.

4. F.L. Lewis and V.L. Syrmos, A geometric theory for derivative feedback," IEEE Trans. Automat. Control, submitted.

5. F.L. Lewis, G. Beauchamp, K. Özçaldiran, and R.P. Malhamé, "Large-scale interconnections of stochastic singular systems," Circuits, Systems, and Signal Proc., submitted.

6. V.L. Syrmos and F.L. Lewis, "A geometric theory of invariant,

partitioned, and deflating subspaces for singular systems," SIAM J. Control and Opt., submitted.

7. A. Karamancioḡlu and F.L. Lewis, "A geometric approach to 2-D singular systems using the Roesser model," in preparation.

Refereed Conference Papers

1. G. Beauchamp and F.L. Lewis, "On the analysis and solution of two-dimensional boundary-value discrete singular systems," Proc. IEEE Conf. Decision and Control, Tampa, FL, Dec. 1989, submitted.

III. RELATED ACTIVITIES OF P.I.

1. Received a Fulbright Award to perform research in singular ssytems at Democritus Univ., Xanthi, Greece, Fall 1988-Spring 1989.

2. Organizer and Joint Chairman, Invited Session on "Singular Systems", Conf. on MTNS, Amsterdam, June 1989.

3. Co-Chairman, Invited Session on "Singular Systems," American Control Conf., June 1989.

4. Member, International Program Committee, IFAC Workshop on System Structure and Control, Prague, Czechoslovakia, Sept. 1989.

5. Organizer and Chairman, Invited Session on "Singular Systems", IFAC Workshop on System Structure and Control, Prague, Czechoslovakia, Sept. 1989.

IV. RELATED PH. D. THESES UNDER SUPERVISION

1. G. Beauchamp, "Structure Algorithms for Singular Systems," expected completion Dec. 1989.

2. D. Fountain, "Geometric Theory for Singular Systems," expected completion 1990.

3. V. Syrmos, "Theory of Singular Systems," expected completion 1991.

4. A. Karamancioglu, "Singular Systems," expected completion 1992.

APPENDIX B
RECENT PAPERS

E-21-675

STRUCTURE AND OUTPUT FEEDBACK

IN SINGULAR SYSTEMS

NSF Grant ECS-8805932

Annual Progress Report for
May 1989 - May 1990

F. L. Lewis
School of Electrical Engineering
Georgia Institute of Technology
Atlanta, GA 30332
404-894-2994

May 8, 1990

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(Sent to NSF under separate cover.)

1. G. Beauchamp, A. Banaszuk, M. Kocięcki, and F.L. Lewis, "Inner and outer geometry for singular systems with computation of subspaces," Int. J. Control, to appear 1991.

2. F.L. Lewis, W. Marszalek, and B.G. Mertzios, "Walsh function analysis of 2-D generalized continuous systems," IEEE Trans. Automat. Control, to appear, 1990.

3. A. Karamancioḡlu and F.L. Lewis, "A geometric approach to singular 2-D systems," Report FLL-90-1, School of Elect. Eng., Ga. Inst. of Technology, Atlanta, GA 30332, Mar. 1990.

4. B.L. Stevens, "Derivation of aircraft linear state equations from implicit nonlinear equations," Proc. IEEE Conf. Decision and Control, Dec. 1990, submitted.

I. INTRODUCTION

Goals

There were two goals in the original proposal. They were to:

1. Study the properties and extensions of the singular system structure algorithm (SSSA)

2. Study proportional-plus-derivative (PD) output feedback in state-variable and singular systems.

We have obtained results in these areas, while also extending the scope of the grant to include some additional goals, namely

3. Develop new geometric techniques for the study of structure in generalized systems

4. Develop tools for the study of structure in 2-D implicit systems

5. Investigate the use of implicit systems in aircraft analysis and design.

Activities and Research

A Summary of Activities and Research appears in Appendix A. Briefly, during this reporting period:

1. Matching funding in the amount of $35,121 was obtained to help support the activities under the NSF grant.

2. One Ph.D. was completed and four are under supervision in this area.

3. Five Research Associates from around the world visited Georgia Tech to perform collaborative research. Moreover, K.M. Przyluski, Polish Academy of Sciences, and J.D. Aplevich, Assoc. Dean, Univ. of Waterloo, will spend sabbaticals at Georgia Tech beginning Sept. 1990.

4. A research monograph is in preparation.

5. Eighteen journal papers and five invited conference papers have appeared or been accepted.

Results

As far as the progress under Goal 1 goes, new results include:

- A geometric characterization of the nullspaces of the matrices on completion of the SSSA.

- A reduced-order technique for the computation of subspaces and the solution of Riccati equations.

Under goal 2, last year we developed a geometric theory for PD state-variable feedback (see last year's annual report). We are now investigating output feedback.

Results under goal 3 include:

- The new notions of outer image and inner preimage of subspaces, and of composite subspaces. A streamlined approach to the classification of subspaces in the domain and codomain, to system properties, and to duality.

Under goal 4, we have:

- A geometric theory for 2-D Roesser implicit systems.

- A geometric approach for observer design for 2-D systems.

- A Walsh approach to the solution of continuous 2-D implicit systems.

Under goal 5, we have:

- A simplified approach to the derivation of linearized aircraft equations of motion based on implicit formulations.

We shall briefly discuss some of the results here. All references are to the papers listed in Appendix A. Some recent relevant papers are enclosed as Appendix B.

II. OUTPUT-NULLING AND COMPOSITE SUBSPACES

Here, we summarize some of the results of [22,24].

To develop a geometric theory that is based firmly on the dynamical properties of singular systems, let us consider the discrete system

$$Ex_{k+1} = Ax_k + Bu_k \tag{2.1}$$
$$y_k = Cx_k + Du_k, \tag{2.2}$$

with $\mathbf{R}^n \equiv \mathbf{X}$, $u \in \mathbf{R}^m \equiv \mathbf{U}$, $y \in \mathbf{R}^p \equiv \mathbf{Y}$, and E a q x n matrix. That is, we shall not assume regularity. We shall call $\mathbf{X}$ the <u>inner space</u>, and $\mathbf{R}^q$, the codomain of E and A, the <u>outer space</u> $\underline{\mathbf{X}}$. Subspaces of $\mathbf{X}$ will be called <u>inner subspaces</u>, and subspaces of $\underline{\mathbf{X}}$ will be underlined and called <u>outer subspaces</u>. The semistate (or <u>inner variable</u>) is x_k while Ex_k and Ax_k are <u>outer variables</u>.

By $\underline{x}_k$ we mean the causal sequence $\{x_0, x_1, \ldots, x_{k-1}\}$. Similarly defined are $\underline{y}_k$ and $\underline{u}_k$. By $\underline{x}'_k$ we mean the possibly noncausal sequence $\{\ldots, x_{-1}, x_0, x_1, \ldots, x_{k-1}\}$. By a <u>trajectory</u>, we shall mean a solution sequence x_k to (2.1), generally noncausal.

For simplicity in this section we assume that D= 0.

Define the subspace sequences:

<u>Unknown-input (UI) unobservable subspace</u>:

$$\mathbf{V}_k = \{x_0 \epsilon\ \mathbf{X} \mid \exists \text{ a trajectory } \underline{x}'_{k+1} \text{ for some } \underline{u}_k \text{ with } \underline{y}_k = 0\}$$

<u>Null-output (NO) reachable subspace</u>:

$$\mathbf{R}_k = \{x_k \epsilon\ \mathbf{X} \mid \exists \text{ a causal trajectory } \underline{x}_{k+1} \text{ for some } \underline{u}_k \text{ with } \underline{y}_k = 0\}$$

<u>Almost reachability subspace (ars)</u>:

$$\mathbf{R}^C{}_k = \{x_k \epsilon\ \mathbf{X} \mid \exists \text{ a causal trajectory } \underline{x}_{k+1} \text{ for some } \underline{u}_k \text{ with } \underline{y}_{k+1} = 0\}$$

By using standard techniques we may show that these subspaces are computed using the following recursions, where **B** indicates the image of B and **N**(C) the nullspace of C:

$$\mathbf{V}_{k+1} = A^{-1}(E\mathbf{V}_k + \mathbf{B}) \cap \mathbf{N}(C) \quad , \quad \mathbf{V}_0 = \mathbf{R}^n \tag{2.3}$$

$$\mathbf{R}_{k+1} = E^{-1}[A(\mathbf{R}_k \cap \mathbf{N}(C)) + \mathbf{B}] \quad , \quad \mathbf{R}_0 = \mathbf{N}(E) \tag{2.4}$$

$$\mathbf{R}^C{}_{k+1} = E^{-1}(A\mathbf{R}^C{}_k + \mathbf{B}) \cap \mathbf{N}(C) \quad , \quad \mathbf{R}^C{}_0 = \mathbf{N}(E) \cap \mathbf{N}(C). \tag{2.5}$$

The limiting values of these recursions will be denoted respectively by $\mathbf{V}_*$, $\mathbf{R}_*$, $\mathbf{R}^C{}_*$.

These three subspace sequences are inner subspaces, residing in $\mathbf{X}$. Define three outer subspace sequences of $\underline{\mathbf{X}}$ by

$$\underline{\mathbf{V}}_k = E\mathbf{V}_k \tag{2.6}$$

$$\underline{\mathbf{V}}^B{}_k = \underline{\mathbf{V}}_k + \mathbf{B} = E\mathbf{V}_k + \mathbf{B} \tag{2.7}$$

$$\underline{\mathbf{R}}_{k+1} = A\mathbf{R}^C{}_k + \mathbf{B} \tag{2.8}$$

It is straightforward to give all of the outer subspaces sensible dynamical definitions in terms of outer variables. The limiting values of these subspaces are again denoted by subscript *.

The next result is a main duality result. Superscript "d" denotes the formal mathematical dual.

Theorem 2.1

1. $\mathbf{V}_k{}^d = \underline{\mathbf{R}}_k{}^{\perp}$
2. $\underline{\mathbf{V}}_k{}^d = \mathbf{R}_k{}^{\perp}$
3. $\underline{\mathbf{V}}^B{}_k{}^d = \mathbf{R}^C{}_k{}^{\perp}$ ∎

It is important to note that the duals of inner subspaces are outer subspaces.

Define the <u>composite image</u> of an inner subspace $\mathbf{S} \subset \mathbf{X}$ as

$$\underline{\mathbf{S}} = \mathbf{ES} + \mathbf{AS} + \mathbf{B} \tag{2.9}$$

and the <u>composite preimage</u> of an outer subspace $\underline{\mathbf{T}} \subset \underline{\mathbf{X}}$ as

$$\mathbf{T} = \mathrm{E}^{-1}\underline{\mathbf{T}} \cap \mathrm{A}^{-1}\underline{\mathbf{T}} \cap \mathbf{N}(\mathrm{C}). \tag{2.10}$$

Then, if $\mathbf{S} = \underline{\mathbf{T}}^{\perp}$, we have $\underline{\mathbf{S}} = \mathbf{T}^{\perp}$. That is, the composite preimage and the composite image are dual concepts.

Note that

1. $\underline{\mathbf{V}}^B{}_* = \mathbf{EV}_* + \mathbf{B} \quad = \quad \mathbf{EV}_* + \mathbf{AV}_* + \mathbf{B}$
2. $\mathbf{V}_* = \mathrm{A}^{-1}\underline{\mathbf{V}}^B{}_* \cap \mathbf{N}(\mathrm{C}) \quad = \quad \mathrm{E}^{-1}\underline{\mathbf{V}}^B{}_* \cap \mathrm{A}^{-1}\underline{\mathbf{V}}^B{}_* \cap \mathbf{N}(\mathrm{C})$
3. $\underline{\mathbf{R}}_* = \mathbf{AR}^C{}_* + \mathbf{B} \quad = \quad \mathbf{ER}^C{}_* + \mathbf{AR}^C{}_* + \mathbf{B}$
4. $\mathbf{R}^C{}_* = \mathrm{E}^{-1}\underline{\mathbf{R}}_* \cap \mathbf{N}(\mathrm{C}) \quad = \quad \mathrm{E}^{-1}\underline{\mathbf{R}}_* \cap \mathrm{A}^{-1}\underline{\mathbf{R}}_* \cap \mathbf{N}(\mathrm{C})$

That is, the subspaces $\mathbf{V}_*$ and $\underline{\mathbf{V}}^B{}_*$ (and also $\mathbf{R}^C{}_*$ and $\underline{\mathbf{R}}_*$) are respectively the composite preimage and composite image of each other.

In the state-space case E= I, if C= 0 then $\mathbf{R}_*$ is the reachable subspace, while if B= 0 then $\mathbf{V}_*$ is the unobservable subspace. However, due to the nonoriented or noncausal nature of singular systems, things are not so simple in case E is singular or nonsquare. Therefore, let us define the composite subspaces

$$\mathfrak{R}_k = \mathbf{V}_* \cap \mathbf{R}^C{}_k \tag{2.11}$$

$$\underline{\mathfrak{R}}_k = \underline{\mathbf{V}}^B{}_* \cap \underline{\mathbf{R}}_k \tag{2.12}$$

$$\mathfrak{S}_k = \mathbf{V}_k + \mathbf{R}^C{}_* \tag{2.13}$$

$$\underline{\mathfrak{S}}_k = \underline{\mathbf{V}}^B{}_k + \underline{\mathbf{R}}_*. \tag{2.14}$$

Note which of these are inner and which outer subspaces.

In the case E= I, $\mathfrak{R}_*$ is the supremal reachability subspace contained in $\mathbf{N}(C)$. In this case, if $C= I$, then $\mathfrak{R}_* = \mathbf{R}_*$ the reachable subspace. However, in the singular case $E \neq I$, even if $C= I$ it is necessary to use the intersection (2.11) to compute the reachable subspace. Thus, the computation of reachable, unobservable, and other subspaces is complicated in the singular case. The theory presented here makes the situation more clear.

We call $\mathfrak{R}_*$ ($\underline{\mathfrak{R}}_*$) the <u>supremal inner (outer) null-output reachability subspace</u>. We call $\mathfrak{S}_*$ ($\underline{\mathfrak{S}}_*$) the <u>infimal inner (outer) unknown-input unobservability subspace</u>. These names are justified in the references in Appendix A.

The next theorem summarizes some of the properties of the composite subspaces.

<u>Theorem 2.2</u>

1. $\mathfrak{R}_k^{\,d} = \underline{\mathfrak{S}}_k^{\perp}$

   $\underline{\mathfrak{R}}_k^{\,d} = \mathfrak{S}_k^{\perp}$

2. $\mathfrak{R}_* = E^{-1}\underline{\mathfrak{R}}_* \cap A^{-1}\underline{\mathfrak{R}}_* \cap \mathbf{N}(C) \quad = E^{-1}\underline{\mathbf{R}}_* \cap A^{-1}\underline{\mathbf{V}}^{B}_* \cap \mathbf{N}(C)$

   $\underline{\mathfrak{R}}_* = E\mathfrak{R}_* + A\mathfrak{R}_* + \mathbf{B} \quad = E\mathfrak{R}_* + \mathbf{B} \quad = A\mathfrak{R}_* + \mathbf{B}$

   $\mathfrak{S}_* = E^{-1}\underline{\mathfrak{S}}_* \cap A^{-1}\underline{\mathfrak{S}}_* \cap \mathbf{N}(C) \quad = E^{-1}\underline{\mathfrak{S}}_* \cap \mathbf{N}(C) \quad = A^{-1}\underline{\mathfrak{S}}_* \cap \mathbf{N}(C)$

   $\underline{\mathfrak{S}}_* = E\mathfrak{S}_* + A\mathfrak{S}_* + \mathbf{B} \quad = E\mathbf{V}_* + A\mathbf{R}^{C}_* + \mathbf{B}$

■

Part 1 demostrates the duality between the composite subspaces. Part 2 demonstrates that the inner and outer versions of the subspaces are the image and preimage of each other, as well as giving simplified expressions for the image of $\mathfrak{R}_*$ and the preimage of $\underline{\mathfrak{S}}_*$.

Based on these definitions, it is shown in [22,24] that a complete geometric theory may be developed for singular systems that is smoother than any presented to date. There, dynamical definitions for all the subspaces are given in terms of system properties.

III. PROPERTIES OF THE SINGULAR SYSTEM STRUCTURE ALGORITHM

The SSSA is given as follows [24].

Algorithm 3.1: (Singular System Structure Algorithm (SSSA))

Step 0. Initialize:

set k= 0.

Define $E_0= E$, $A_0= A$, $B_0= B$, $C_0= C$, $D_0= D$, $\underline{C}_0= 0$, $W_0= 0$.

Step 1. Iteration k:

Find constant unitary linear transformations T_k and S_k such that

$$T_k \begin{array}{c} \begin{array}{ccc} n & n & m \end{array} \\ \left[\begin{array}{ccc} E_k & A_k & B_k \\ \underline{C}_k & 0 & 0 \end{array}\right] \end{array} \begin{array}{c} \\ r_k \\ t_k \end{array} = \begin{array}{c} \begin{array}{ccc} n & n & m \end{array} \\ \left[\begin{array}{ccc} E_{k+1} & A_{k+1} & B_{k+1} \\ 0 & \underline{A}_k & \underline{B}_k \end{array}\right] \end{array} \begin{array}{c} \\ r_{k+1} \\ \end{array}$$

$$S_k \begin{array}{c} \begin{array}{cc} n & m \end{array} \\ \left[\begin{array}{cc} C_k & D_k \\ \underline{A}_k & \underline{B}_k \end{array}\right] \end{array} \begin{array}{c} \\ s_k \\ \end{array} = \begin{array}{c} \begin{array}{cc} n & m \end{array} \\ \left[\begin{array}{cc} C_{k+1} & D_{k+1} \\ \underline{C}_{k+1} & 0 \end{array}\right] \end{array} \begin{array}{c} \\ s_{k+1} \\ t_{k+1} \end{array}$$

with both E_{k+1} and D_{k+1} having full row rank r_{k+1} and s_{k+1} respectively for all $k \geq 0$.

Define

$$W_{k+1} = \begin{bmatrix} W_k \\ \underline{C}_{k+1} \end{bmatrix}$$

Stopping Criterion:

Stop if $\text{rank}(W_{k+1}) = \text{rank}(W_k)$. Then, set L= k, End. Else, set k= k+1, go to Step 1. ■

The SSSA has been used to compute subspaces for singular systems. In fact,

$$\mathbf{N}(W_L) = \mathbf{V}_*. \tag{3.1}$$

It has also been shown recently [1] that

$$\mathbf{N}(E_L) = \mathbf{N}(E) \cap \mathbf{V}_* \tag{3.2}$$

$$\mathbf{N}(D_L) = B^{-1}E\mathbf{V}_* \cap \mathbf{N}(D). \tag{3.3}$$

These results open the way for the use of the SSSA in the inversion of implicit systems and elsewhere. The ramifications are being explored.

In [24] is shown a simplified technique for solving Riccati equations that only involves solving a (nonsquare) Lyapunov equation on a subspace. This applies to the usual Riccati equations in optimal control and estimation and offers significant computational savings.

IV. 2-D IMPLICIT SYSTEMS

A generalization of the Roesser 2-D model is

$$\begin{bmatrix} E_1 & E_2 \\ E_3 & E_4 \end{bmatrix} \begin{bmatrix} x^h_{i+1,j} \\ x^v_{i,j+1} \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} x^h_{i,j} \\ x^v_{i,j} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u_{i,j} \tag{4.1}$$

$$y_{i,j} = [C_1 \quad C_2] \begin{bmatrix} x^h_{i,j} \\ x^v_{i,j} \end{bmatrix} + Du_{i,j}\ . \tag{4.2}$$

By appropriate definition of variables we may write this more compactly as

$$\tilde{E}x_{i,j} = Ax_{i,j} + Bu_{i,j} \tag{4.3}$$

$$y_{i,j} = Cx_{i,j} + Du_{i,j}\ . \tag{4.4}$$

We call $x^h_{i,j} \in \mathbf{R}^{n_1}$ the horizontal semistate, $x^v_{i,j} \in \mathbf{R}^{n_2}$ the vertical semistate and $x_{i,j} \in \mathbf{R}^n$ (with $n = n_1 + n_2$) the local semistate.

The implicit 2-D models are more useful than the state-space models, where E= I, since they may describe nonrecursible masks, as well as discretized versions of the heat equation (with boundary conditions on all sides of a region), and the hyperbolic equation. These cannot be described by state-space 2-D models, which require quarter-plane causality.

Define

$$E_I = \begin{bmatrix} E_1 & 0 \\ E_2 & 0 \end{bmatrix} \quad , \quad E_{II} = \begin{bmatrix} 0 & E_3 \\ 0 & E_4 \end{bmatrix} \tag{4.5}$$

and similar quantities with respect to A. Then (4.1) may be written as

$$E_I x_{i+1,j} + E_{II} x_{i,j+1} = A x_{i,j} + B u_{i,j}. \tag{4.6}$$

Alternatively, in terms of the semistate $\tilde{x}_{i,j}$ with shifted horizontal and vertical components, one may write (4.1) as

$$E\tilde{x}_{i,j} = A_I \tilde{x}_{i-1,j} + A_{II} \tilde{x}_{i,j-1} + B u_{i,j}. \tag{4.7}$$

We have defined an (A,E,**B**)-controlled invariant for the 2-D implicit system as a subspace **V** satisfying

$$A_I \mathbf{V} + A_{II} \mathbf{V} = E\mathbf{V} + \mathbf{B} \tag{4.8}$$

and an (E,A,**B**)-controlled invariant as one satisfying

$$E_I \mathbf{V} + E_{II} \mathbf{V} = A\mathbf{V} + \mathbf{B}. \tag{4.9}$$

In terms of these subspaces, we are developing a geometric theory for the implicit Roesser models.

In [Karamancioḡlu and Lewis 1990] (enclosed) we have used a geometric approach to design observers for the 2-D implicit Fornasini-Marchesini (FM) model

$$E x_{i+1,j+1} = F x_{i+1,j} + G x_{i,j+1} + B_1 u_{i+1,j} + B_2 u_{i,j+1} \tag{4.10}$$

$$y_{i,j} = C_1 x_{i+1,j} + C_2 x_{i,j+1}. \tag{4.11}$$

The technique relies on solving a 2-D Sylvester equation.

We have given results for the analysis of both the Roesser and FM implicit models in several references (see Appendix A). In [1] we considered the FM model. In [23] we used the <u>fundamental matrix sequence</u> to analyze 2-D systems. In [15] we used Walsh functions to solve the continuous version of (4.1), which is a two-variable partial differential equation. Currently, one Ph.D. student is working on 2-D implicit systems.

V. IMPLICIT EQUATIONS IN AIRCRAFT MODELING

In [Stevens 1990] (enclosed), we have shown that the linearized aircraft equations of motion may be very conveniently derived using an implicit formulation. This approach is far simpler than the traditional ones.

The aircraft nonlinear equations are of the implicit form

$$F(X,\dot{X},X)= 0 \tag{5.1}$$

with state vector

$$X= [v_T \quad \alpha \quad \beta \quad \phi \quad \theta \quad \psi \quad p \quad q \quad r]^T, \tag{5.2}$$

where v_T is total velocity, α and β are bank angle and sideslip, ϕ, θ, ψ are roll, pitch, and yaw, and the last three states are roll-rate, pitch-rate, and yaw-rate.

The nonlinear equations consist of 3 force equations, 3 kinematic equations, and 3 moment equations.

By simply linearizing (5.1) according to

$$-\frac{\partial F}{\partial \dot{X}}\dot{x} = \frac{\partial F}{\partial X}x + \frac{\partial F}{\partial U}u \tag{5.3}$$

we obtain implicit linear equations of the form

$$E\dot{x}= Ax + Bu. \tag{5.4}$$

Then, transfer functions to various outputs defined by

$$y= Cx \tag{5.5}$$

may be directly obtained using

$$Y(s)= C(sE-A)^{-1}BU(s). \tag{5.6}$$

Although this is a standard approach in implicit systems, it is not traditionally used in aircraft analysis. It does considerably simplify the derivation of small perturbation transfer functions, while also giving more accurate results.

To illustrate the sort of problem that can occur in aircraft modeling, in the force equations we have

$$
\begin{aligned}
m\dot{v}_T \quad &+ \quad + mg_1 + D - F_T \cos\alpha \cos\beta = 0 \\
mv_T\dot{\beta} \quad &+ mv_T r + mg_2 + Y + F_T \cos\alpha \sin\beta = 0 \qquad (5.7) \\
mv_T \cos\beta\, \dot{\alpha} &+ mv_T q + mg_3 + L + F_T \sin\alpha \quad = 0,
\end{aligned}
$$

where lift L is a function of $\dot{\alpha}$. However, $\dot{\alpha}$ is not a state, but α is. This is not easily handled using standard approaches in aircraft modeling, but using our approach all that happens is that the E matrix in (5.4) is not the identity.

REDUCED BY 2% for Gram-Rudman

YEAR 3

SUMMARY PROPOSAL BUDGET

OMB No. 3145-0058
Exp. Date 12/31/86

FOR NSF USE ONLY		
PROPOSAL NO.	DURATION (MONTHS)	
	Proposed	Granted
AWARD NO.		

ORGANIZATION
GEORGIA TECH RESEARCH CORPORATION

PRINCIPAL INVESTIGATOR/PROJECT DIRECTOR
F. L. LEWIS

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title; A.6. show number in brackets)	NSF FUNDED PERSON-MOS. CAL.	ACAD	SUMR	FUNDS REQUESTED BY PROPOSER	FUNDS GRANTED BY NSF (IF DIFFERENT)
1. F. L. LEWIS		2.25	1.0	$ 17,855	$
2.					
3.					
4.					
5. ( ) OTHERS (LIST INDIVIDUALLY ON BUDGET EXPLANATION PAGE)					
6. (1) TOTAL SENIOR PERSONNEL (1-5)		2.25	1.0	17,855	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. ( ) POST DOCTORAL ASSOCIATES					
2. ( ) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)					
3. (2) GRADUATE STUDENTS (PhD at 1/3 time ea. for 4 qtrs.)				19,051	
4. ( ) UNDERGRADUATE STUDENTS					
5. ( ) SECRETARIAL-CLERICAL					
6. ( ) OTHER					
TOTAL SALARIES AND WAGES (A+B)				36,906	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS) (27.6% of A6)				4,928	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A+B+C)				41,834	
D. PERMANENT EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $1,000:)					
TOTAL PERMANENT EQUIPMENT					
E. TRAVEL 1. DOMESTIC (INCL. CANADA AND U.S. POSSESSIONS) 1397				~~1,500~~	
2. FOREIGN				1,500	
F. PARTICIPANT SUPPORT COSTS					
1. STIPENDS $					
2. TRAVEL					
3. SUBSISTENCE					
4. OTHER					
TOTAL PARTICIPANT COSTS					
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES					
2. PUBLICATION COSTS/PAGE CHARGES				~~300~~	
3. CONSULTANT SERVICES				~~1,000~~	
4. COMPUTER (ADPE) SERVICES					
5. SUBCONTRACTS					
6. OTHER					
TOTAL OTHER DIRECT COSTS				~~1,300~~	
H. TOTAL DIRECT COSTS (A THROUGH G) 44,731				~~46,134~~	
I. INDIRECT COSTS (SPECIFY) Proposed rate for the period 7/1/87-6/30/88 and subject to change. (60.0% of H) TOTAL INDIRECT COSTS				27,680	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I) 72,411				~~73,814~~	
K. 5% Cost-Sharing by the School of Electrical Engineering				3,691	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K) 68,720				$ ~~70,123~~	$

PI/PD TYPED NAME & SIGNATURE*	DATE	FOR NSF USE ONLY		
F. L. LEWIS	11/24/87	INDIRECT COST RATE VERIFICATION		
INST. REP. TYPED NAME & SIGNATURE*	DATE	Date Checked	Date of Rate Sheet	Initials DGC
LYNN BOYD				Program

NSF Form 1030 (4-83) *Supersedes All Previous Editions*

*SIGNATURES REQUIRED ONLY FOR REVISED BUDGET (GPM 233)

APPENDIX A
SUMMARY OF ACTIVITIES AND RESEARCH

I. RELATED GRANTS AND PAPERS

Related Grants

F.L. Lewis, "International cooperative program in singular systems," Georgia Tech Foundation Grant, $9000, Mar. 1990.

F.L. Lewis, "Matching funds for NSF grant ECS-8805932," $26,121, Aug. 1989.

Books in Preparation

K. Özçaldiran and F.L. Lewis, Singular Systems, Prentice-Hall, International Series in Systems and Control Engineering, contract under negotiation, to appear, 1991.

Ph.D. Theses

1. G. Beauchamp, Algorithms For Singular Systems, Ph.D. Thesis, School of Electrical Engineering, Georgia Institute of Technology, Atlanta, GA 30332, Mar. 1990.

Invited and Solicited Conference Papers

2. F.L. Lewis, G. Beauchamp, and V.L. Syrmos, "Some useful aspects of the infinite structure in singular systems," Proc. Int. Symp. MTNS, vol. 1, June 1989.

3. F.L. Lewis and D.W. Fountain, "Generalized notions in geometry and duality," Proc. American Control Conf., pp. 2146-2151, June 1989.

4. G. Beauchamp, F.L. Lewis, and B. Mertzios, "Recent results in 2-D singular systems," Proc. IFAC Workshop on System Structure and Control, pp. 253-256, Prague, Czechoslovakia, Sept. 1989.

5. F.L. Lewis, "A tutorial on the properties of linear singular systems," IFAC Workshop on System Structure and Control, Prague, Czechoslovakia, Sept. 1989.

6. F.L. Lewis and D.W. Fountain, "Some singular systems applications in circuit theory," Proc. International Symposium on Circuits and Systems, New Orleans, May 1990.

Journal Papers

7. K. Özçaldiran and F.L. Lewis, "Generalized reachability subspaces for singular systems," SIAM J. Control and Opt., vol. 27, no. 3, pp. 495-510, May 1989.

8. F.L. Lewis, M.A. Christodoulou, B.G. Mertzios, and K. Özçaldiran, "Chained aggregation of singular systems," IEEE Trans. Automat. Control, vol. 34, no. 9, pp. 1007-1013, Sept. 1989.

9. B.G. Mertzios and F.L. Lewis, "Fundamental matrix of discrete singular systems," Circuits, Systems, and Signal Proc., vol. 8, no. 3, pp. 341-355, 1989. **Invited Paper.**

10. F.L. Lewis, "Computational geometry for design in singular systems," IMACS Annals Comp. and Appl. Math., vol. 3, Modeling and Simulation of Systems, P. Breedveld et al., ed., pp. 381-383, Dec. 1989. **Solicited Paper.**

11. F.L. Lewis, V.G. Mertzios, G. Vachtsevanos, and M.A. Christodoulou, "Analysis of bilinear systems using Walsh functions," IEEE Trans. Automat. Control, vol. AC-35, no. 1, pp. 119-123, Jan. 1990.

12. F.L. Lewis and B.G. Mertzios, "On the analysis of discrete linear time-invariant singular systems," IEEE Trans. Automat. Control, vol. 35, no. 4, pp. 506-511, April 1990.

13. F.L. Lewis, "Geometric design techniques for observers in singular systems," Automatica, to appear, 1990.

14. K. Özçaldiran and F.L. Lewis, "On the regularizability of singular systems," IEEE Trans. Automat. Control, to appear, 1990.

15. F.L. Lewis, W. Marszalek, and B.G. Mertzios, "Walsh function analysis of 2-D generalized continuous systems," IEEE Trans. Automat. Control, to appear, 1990.

16. F.L. Lewis, G. Beauchamp, K. Özçaldiran, and R.P. Malhamé, "Large-scale dynamical interconnections of stochastic singular systems," Circuits, Systems, and Signal Proc., to appear, 1990.

17. F.L. Lewis, B.G. Mertzios, and W. Marszalek, "Analysis of singular bilinear systems using Walsh finctions," IEEE Proc.- Part D, to appear, 1990.

18. V.L. Syrmos and F.L. Lewis, "A geometric approach to proportional-plus-derivative feedback using quotient and partitioned subspaces," Automatica, to appear, 1990.

19. F.L. Lewis, "An introduction to 2-D implicit systems," IMACS Annals Comp. and Appl. Math., W.H. Wimmers ed., to appear 1990. **Solicited Paper.**

20. A. Banaszuk, M. Kocięcki, G. Beauchamp, and F.L. Lewis, "Observability with unknown input and dual properties for singular systems," IMACS Annals Comp. and Appl. Math., W.H. Wimmers ed., to appear 1990. **Solicited Paper.**

21. F.L. Lewis and V.L. Syrmos, "A geometric theory for derivative feedback," IEEE Trans. Automat. Control, to appear, April 1991.

22. F.L. Lewis, "A tutorial on the geometric properties of linear time-invariant singular systems," Automatica, to appear 1991.

23. F.L. Lewis and B.G. Mertzios, "On the analysis of two-dimensional discrete singular systems," Circuits, Systems, and Signal Proc., to appear 1991.

24. G. Beauchamp, A. Banaszuk, M. Kocięcki, and F.L. Lewis, "Inner and outer geometry for singular systems with computation of subspaces," Int. J. Control, to appear 1991.

II. PH.D. THESES COMPLETED AND UNDER SUPERVISION

1. G. Beauchamp, Algorithms For Singular Systems, Mar. 1990.

2. D. Fountain, Computational Techniques For Implicit Systems, to be completed, Winter 1991.

3. V. Syrmos, Geometric Tools For Analysis and Design in Implicit Systems, to be completed, Spring 1991.

4. A. Karamancioğlu, Two Dimensional Implicit Systems, to be completed, Spring 1991.

5. A. Lowe, Design Techniques For Implcit systems, to be completed, Summer 1991.

III. RESEARCH ASSOCIATES SUPPORTED

1. K. Özçaldiran, Bosporus Univ., Istanbul, Turkey, "Observability and duality in singular systems," Winter 1990.

2. R.P. Malhamé, Ecole Polytechnique de Montréal, Canada, "Markov renewal analysis of flexible manufacturing systems," Winter 1990.

3. S.L. Campbell, Dept. of Math., N. C. State Univ., "Time-varying singular systems," Winter 1990.

4. M. Kocięcki, Warsaw Univ. of Technology, "Geometry of singular systems," Spring 1990.

5. A. Banaszuk, Warsaw Univ. of Technology, "Geometry of singular systems," Spring 1990.

APPENDIX B. RECENT PAPERS (ENCLOSED)